# Properties of Nuclear Matter and Three-Body Forces

***Dr. A. M. Alakrmi***
*Department of Physics,*
*Faculty of Education,*
*Zawia University*

***Dr. A. M. Algamoudi***
*Department of Physics,*
*Faculty of Science,*
*Aljapal Algharbi University*

**ABSTRACT:**

Results of cooled and hot symmetric nuclear matter calculations are presented. The Brueckner – Hartree –Fock (BHF) approximation plus two body density dependent Skyrme potential which is equivalent to three body interaction are used. Argonne $v_{18}$ nucleon-nucleon (NN) potential is used in the framework of (BHFA) .

The bulk properties of symmetric nuclear matter are computed such as the EOS at (T = 0, 8, 12 MeV), pressure at (T = 0, 8, 12 MeV), nuclear matter incompressibility and the symmetry energy. The results are compared with M. Baldo and L. S. Ferreira (year) (BL) calculation.

Good agreement is obtained in comparison with previous theoretical estimates and experimental data.

***Key Words:*** *symmetric nuclear matter- (BHFA) - EOS - symmetry energy.*

# INTRODUCTION:

On a microscopic basis the equation of state (EOS) of symmetric nuclear matter has been extensively studied within the variational approach [1–3] as well as relativistic [4–10] and non relativistic [11, 12] Brueckner–Hartree–Fock (BHF) theories. The predictions of non relativistic microscopic approaches (including both the BHF and variational approaches) based on pure two-body nucleon–nucleon (*NN*) forces (2BF) do not give the empirical saturation point of symmetric nuclear matter (Coester band [13]). In order to improve the nuclear saturation, two lines have been followed. One is the development of the relativistic mean field (RMF) theory [14] and Dirac–Brueckner–Hartree–Fock (DBHF) approach [5, 15–19]. The DBHF has been successful in describing the saturation properties of symmetric nuclear matter (SNM), however, still there are some problems remaining unsettled, such as the negative energy state problem, the ambiguities related to the decomposition of the effective

reaction matrix into covariant amplitudes due to various approximations introduced for reducing the four-dimensional Bethe–Salpeter equation to the corresponding three-dimensional one. In the second line the medium effects are taken into account by phenomenological or microscopic three-body forces (3BF) within non-relativistic contexts. Calculations with phenomenological 3BF have been performed both in the framework of the variational approach [1, 2] and the BHF approximation [20–23]. The basic input quantity in the BHF calculation is the *NN* interaction in free space. In the previous work [24] using BHF we adopted the modern Argonne $v_{18}$ potential [26], and charge-dependent Bonn potential (CD-Bonn) [27]. The recent versions of

The Nijmegen group are Nijm-I, Nijm-II, and Reid93 potentials. In the present work we add the corrections of the three-body forces using an equivalent density dependent two body forces of Skyrme type. Hot systems are also considered for small temperatures. In the next section we give a brief description of the method of calculation. Section 4 is devoted for a presentation of our main results.

## THEORY:

Here we start with a short review of the theoretical framework:

The microscopic Brueckner–Bethe–Goldstone description of nuclear matter is based on a linked cluster expansion of the energy per nucleon of nuclear matter [28].

The basic ingredient is the Brueckner reaction matrix $G$, which is the solution of the Bethe–Goldstone equation :

$$G(\omega) = V + V \frac{Q}{\omega - H_\circ + i\eta} G(\omega). \qquad (1)$$

Here, $\omega$ is the starting energy which is usually the sum of the single-particle energies of the states of the interacting nucleon

$$\omega = e(k) + e(k'). \qquad (2)$$

$V$ is the bare $NN$ potential, $\eta$ is an infinitesimal small number, $H_o$ is the unperturbed energy of the intermediate scattering states, $e$ is the single-particle energy, and $Q$ is the Pauli projection operator; it projects out states with two nucleons above the Fermi level, it is given by:

$$Q(k, k') = (1 - \Theta_f(k))(1 - \Theta_f(k')), \qquad (3)$$

where $\Theta_f(k) = 1$ for $k < k_f$ and zero otherwise, $\Theta_f(k)$ is the occupation probability of a free Fermi gas with Fermi momentum $k_f$

In the Brueckner–Goldstone expansion, the average binding energy per nucleon is expanded in a series of terms as the following:

$$\frac{E(k)}{A} = \langle \hat{T} \rangle + \langle \hat{G} \rangle = \sum_{k} \frac{\hbar^2 k^2}{2m} + \frac{1}{2} \sum_{k,k'<k_f} \langle kk'| G(e(k)+e(k')) | kk' \rangle, \quad (4)$$

where |kk' > refer to antisymmetrized two-body states. This first order is known as the Brueckner–Hartree–Fock (BHF) approximation. To completely determine the average binding energy one has to define the single-particle potential *U (k)* which contributes to the single-particle energies appearing in the *G*-matrix elements. The structure of the expression (4) suggests choosing the following BHF single-particle potential

$$U(k) = \sum_{k'<k_f} \langle kk'| G(e(k)+e(k')) | kk' \rangle \quad (5)$$

$$\frac{E(k)}{A} = \sum_{k<k_f} \left\{ \frac{\hbar^2 k^2}{2m} + \frac{1}{2} U(k) \right\} = \frac{4}{\rho} \int_0^{k_f} \frac{4\pi k^2}{(2\pi)^3} \left( \frac{\hbar^2 k^2}{2m} + \frac{1}{2} U(k) \right) dk =$$

$$\frac{3\hbar^2 k_f^2}{10m} + \frac{3}{2k_f^3} \int_0^{k_f} k^2 dk U(k), \quad (6)$$

where ρ is the matter density.The G-matrix itself depends on *U (k)* through the starting energy *ω*, defined in Eq. (2), and the lowest-order approximation (4) along with choice (5) for the single-particle potential is often known as the lowest-order Brueckner theory. The single particle energy *e (k)* is defined as .

$$e(k) = T + U(k) = \frac{\hbar^2 k^2}{2m} + U(k), \quad (7)$$

where T is the kinetic energy. The conventional choice for the single-particle potential has been to take the BHF potential (Eq. (5)) for hole states ($k < k_f$) and zero for particle states ($k > k_f$), thus introducing a

$$U(k) = \begin{cases} \sum_{k' \leq k} \langle kk' | G(e(k) + e(k')) | kk' \rangle & k \leq k_f \\ 0 & k > k_f \end{cases} \qquad (8)$$

Eqs. (1) and (7) represent the main equations that one has to solve self-consistently. In order to achieve saturation in nuclear matter one has to add three-body interaction terms or a density-dependent two-nucleon interaction. We have chosen it following the notation of the Skyrme interaction to be of the form

$$v(\mathbf{r}_1, \mathbf{r}_2) = \sum_i t_i\, (1 + y_i p_\sigma) \rho^{\alpha_i} \delta(\mathbf{r}_1 - \mathbf{r}_2), \qquad (9)$$

where $\mathbf{r}_1$ and $\mathbf{r}_2$ are the position vectors for the particle 1 and particle 2 respectively, $P_\sigma$ is the spin exchange operator, $\rho$ is the matter density $t_i$ ,$y_i$, and $\alpha_i$ are parameters. For various values of $\alpha_i$ (typically $\alpha_i$ = 1/3, 2/3, 0.5, and 1) we have fitted $t_i$ and $y_i$ in such a way that a

BHF calculation plus the contact terms yield the empirical saturation point for symmetric nuclear matter. Having obtained the energy per particle $E/A$ for zero temperature, the free energy $F = E/A - aT^2$ and the pressure may be calculated at temperature $T$ using the expression of the level density [29]. Among the different sets of parameters $\alpha_i$ proposed here the best results were obtained for two terms of the above summation where $\alpha_1$ and $\alpha_2$ are equal to 1/3 and 2/3 respectively.

## Results and discussion:

### 1. Calculation of the EOS:

The EOS is the relationship between energy per nucleon and Fermi momentum $k_F$ or density, the minimum point of the EOS curve is called the saturation point. The results are shown in the Fig. 1 at $T$=0, 8 and 12 MeV where the energy per particle ($F / A$) in MeV plotted against density ρ in $fm^{-3}$, for symmetric nuclear matter using Argonne $v_{18}$ potential and the parameters of the contact potential are given in table (1). A comparison is made with M. Baldo and L. S. Ferreira (BL) calculation [30] v 14+TNI realistic potential. The results are identical with BL at small densities.

***Table. (1):*** *Interaction Parameters of Argonne $v_{18}$ potential:*

| $t_1$ | $t_2$ | $y_1$ | $y_2$ |
|---|---|---|---|
| -1168.6 | 1887.6 | 0.6643 | -0.2168 |

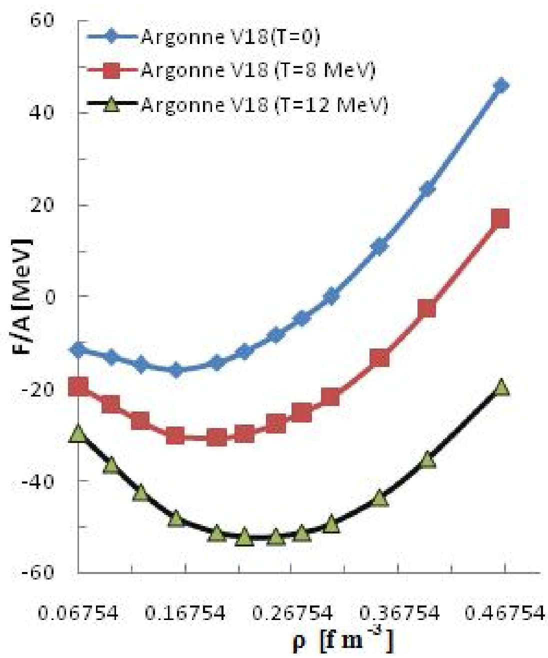


***Fig. 1.*** *F/A of symmetric nuclear matter as a function of density at (T=0, 8 and 12 MeV using Argonne $v_{18}$ potential.*

## 2. Calculation of the free energy:

The free energy of the nuclear matter is defined by:

$$F = E_{T=0} - a T^2 \qquad (10)$$

$$a = (\pi^2/2)(m^*/\hbar^2 k^2_F) \qquad (11)$$

where *F* is the free energy of the system, $E_{T=0}$ is the total energy at *T*=0, and a is the level density of the system. where *m** is the effective mass of the nucleon. . The results are shown in the Fig. 2 at *T*=0 in comparison with M. Baldo and L. S. Ferreira (BL) calculation [30].

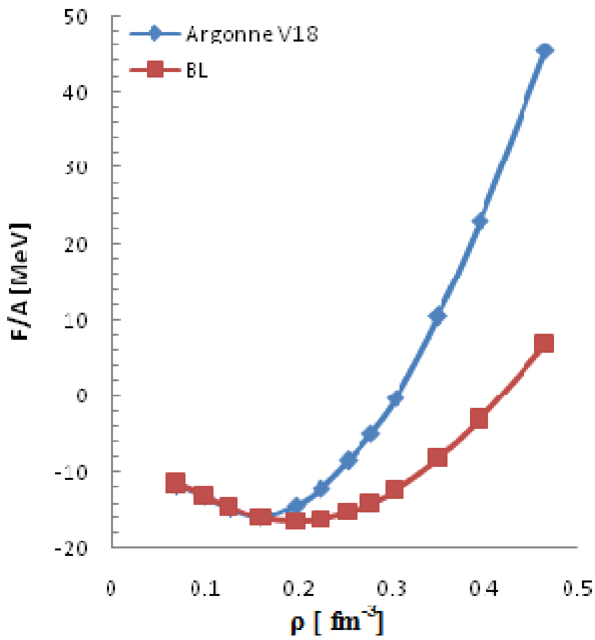


***Fig. 2.*** *F / A in MeV for symmetric nuclear matter at (T=0) as a function of density using Argonne* $v_{18}$ *potential in comparison with BL calculation [30].*

# 3. The pressure:

The pressure for symmetric nuclear matter at $T$=0 is defined in terms of the energy per particle as:

$$P(\rho) = \rho^2 \frac{\partial (E/A)(\rho)}{\partial \rho} \qquad (12)$$

The results are shown in fig. 3, where the values of the pressure at (T=0) are plotted against the density ρ for symmetric nuclear matter using the Argonne $v_{18}$ potential.

At $T$=8 and 12 MeV we have used the equation (12) in equation (10) for calculating the pressure, The results are shown in Figs. 4 and 5, where the values of the pressure at $T$=8 and 12 MeV are plotted against the density ρ for symmetric nuclear matter. The results are very close to the BL calculation at small densities. using the Argonne $v_{18}$ in comparison with BL calculation. Satisfactory agreement is obtained getting the same shape and comparable values with the realistic potential calculation at zero temperature.

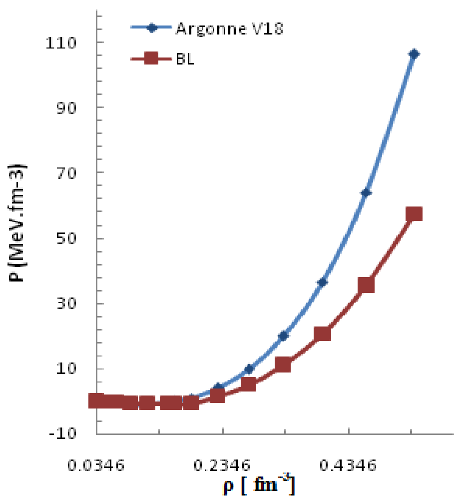


***Fig. 3.** The pressure of symmetric nuclear matter at (T = 0) as a function of density using Argonne $v_{18}$ potential in comparison with BL calculation [30].*

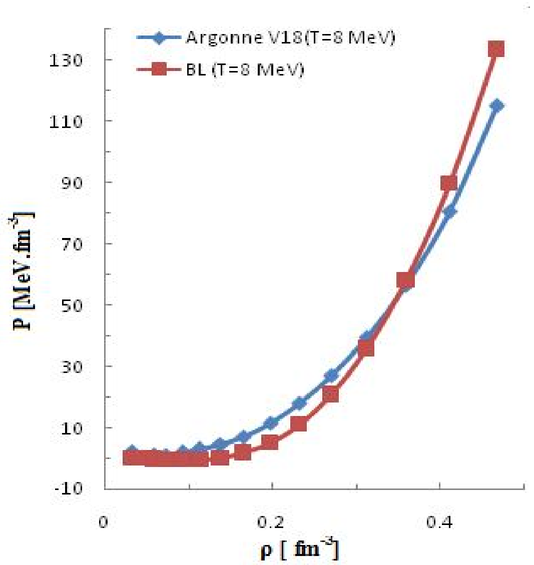


***Fig. 4.*** *The pressure of symmetric nuclear matter at (T =  8 MeV) as a function of density using Argonne $v_{18}$  potential for  in comparison with BL calculation [30].*

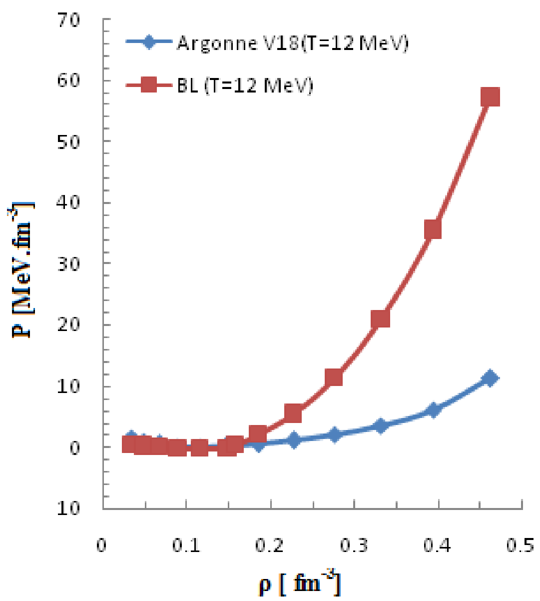


***Fig. 5.*** *The pressure of symmetric nuclear matter at (T = 12 MeV) as a function of density using Argonne $v_{18}$  potential for  in comparison with BL calculation [30].*

## 4. Symmetry energy:

The symmetry energy is defined as:

$$\varepsilon_{\tau}(\rho) = \frac{1}{2}\left[\frac{\partial^2 E/A(\rho,\alpha_{\tau})}{\partial \alpha_{\tau}^2}\right]_{\alpha_{\tau}=0}, \qquad (13)$$

where $\alpha_\tau$ is neutron excess parameter.

In Fig. 6 , the symmetry energy in MeV is plotted against the density in[ $fm^{-3}$], and compared with the experimental data [31] using the Argonne $v_{18}$ potential. The calculations yield similar results with the experimental data at all values of the density. From the fig. 6, one can see that the nuclear symmetry energy increases with increasing the density.

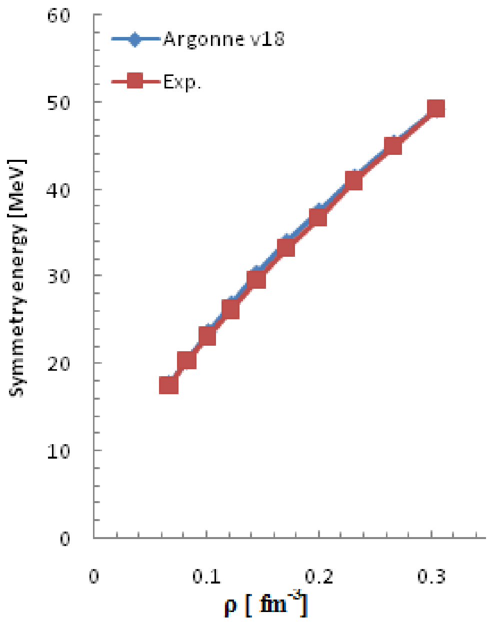


***Fig. 6.*** *The symmetry energy in [MeV] as function of density in [$fm^{-3}$] is compared by exp. data [31] using Argonne $v_{18}$ potential.*

## 5. Nuclear matter incompressibility:

The incompressibility $\kappa_0$ [32] can be calculated from the following equation:

$$\kappa_0 = k_F^2 \frac{\partial^2 (E/A)(k_F)}{\partial k_F^2}\Big|_{k_F = k_F^0} = 9\rho^2 \frac{\partial^2 (E/A)(\rho)}{\partial \rho^2}\Big|_{\rho = \rho_0} \qquad (14)$$

The incompressibility $\kappa_0$ can be used to explain the stiffness of the EOS. The experimental value of the incompressibility of nuclear matter at its saturation density $\rho_0$ has been determined to be 210 ± 30 MeV [33] the incompressibility $\kappa_0$ is calculated with a $4^{-th}$ degree polynomial. In fig. (7), $\kappa_0$ in MeV is plotted against the density in [$fm^{-3}$] using the potential Argonne $v_{18}$ potential.

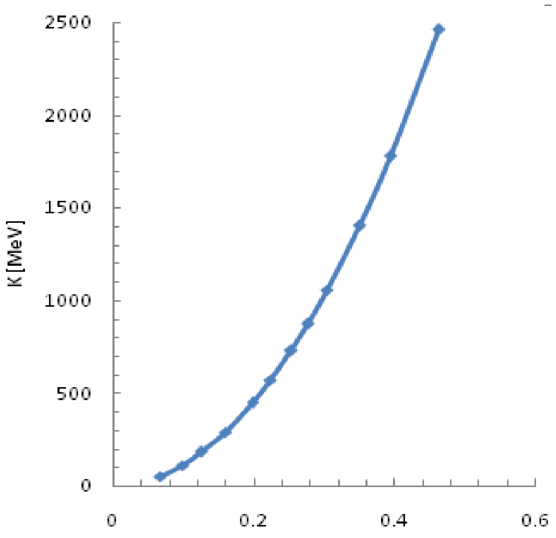


***Fig. 7.*** *The nuclear matter incompressibility κ (ρ) in MeV as a function of density using Argonne $v_{18}$ potential.*

## Conclusion:

This paper presented a calculation of the EOS for symmetric nuclear matter, the symmetry energy, and the pressure of nuclear matter at zero and finite small temperatures. The results are obtained by adding a density-dependent two-body potential to the BHF calculation.

Modern *NN* interactions as the Argonne $v_{18}$ potential is used in the framework of BHF approximation. The results are compared to the $v_{14}$+TNI realistic potential calculation of B L. we conclude that the BHF theory in addition to our suggested contact interaction is able to produce the experimental saturation point for the equation of state and overall good agreement with the realistic force calculation of BL for $T$=0. Good agreement is obtained for the energy per particle, pressure, free energy, and the symmetry energy with the theoretical BL [30] and experimental data Shetty *et al* [31].

Comparable results are obtained for finite temperatures. Two terms are used only in our suggested potential but one can add other terms to calculate other physical quantities. In this case the treatment will be more involving.

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